

Тема 14. Уравнения и неравенства.**Задание 1.** Решите уравнение:

Образец: $x^3 + 4x^2 = 9x + 36;$

$x^3 + 4x^2 - 9x - 36 = 0;$

$x^2(x + 4) - 9(x + 4) = 0;$

$(x^2 - 9)(x + 4) = 0;$

$(x - 3)(x + 3)(x + 4) = 0;$

$x - 3 = 0$ или $x + 3 = 0$ или $x + 4 = 0$

$x = 3$ $x = -3$ $x = -4$

Ответ: -4; -3; 3.

1) $x^3 + 7x^2 = 4x + 28;$

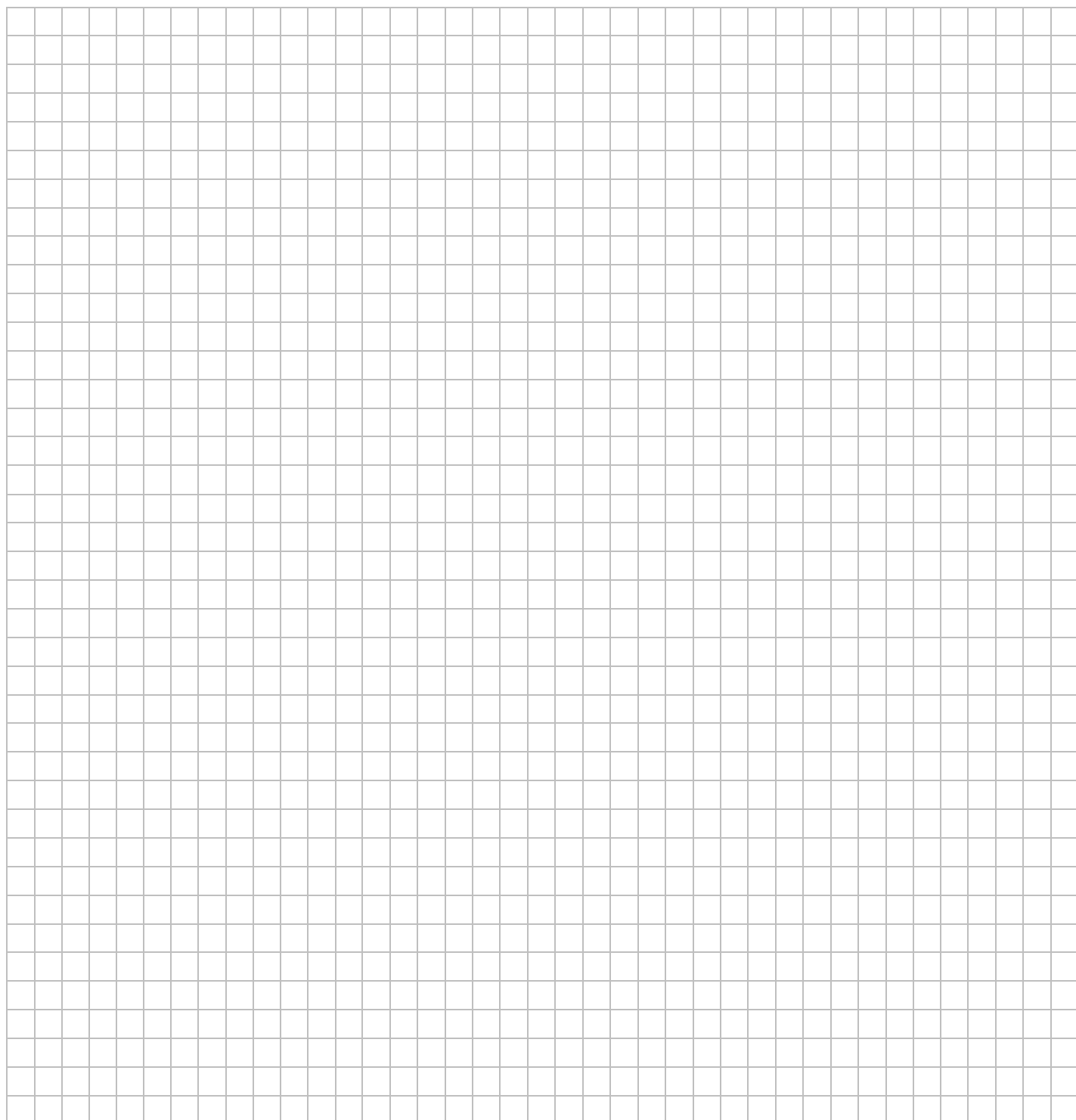
2) $x^3 + 5x^2 - x - 5 = 0;$

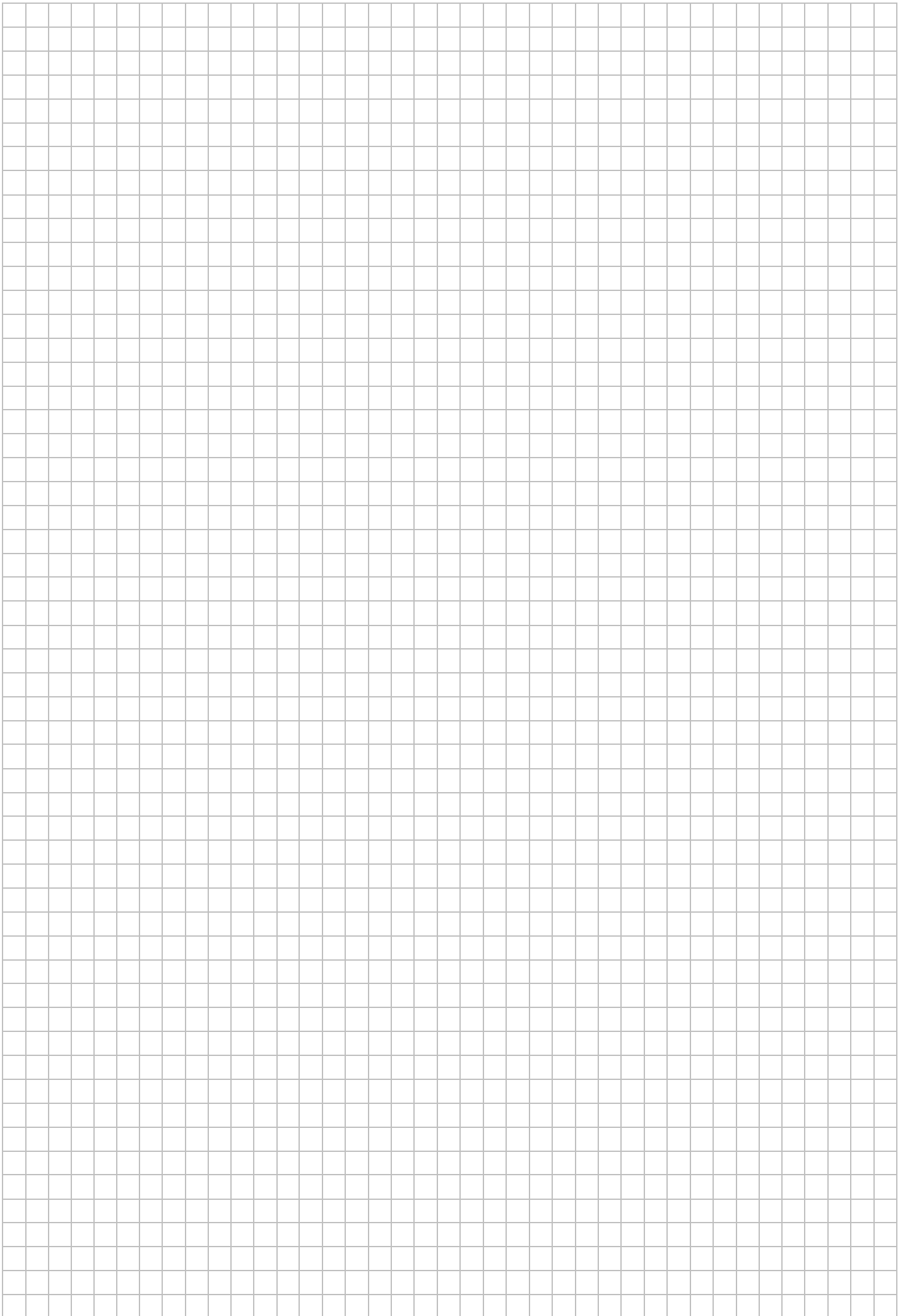
3) $x^3 + 2x^2 = 9x + 18;$

4) $x^3 + 2x^2 - x - 2 = 0;$

5) $x^3 + x^2 = 16x + 16;$

6) $x^3 + 3x^2 - 4x - 12 = 0.$





Задание 2. Решите уравнение:

Образец: $x^2 - 2x + \sqrt{3-x} = \sqrt{3-x} + 8;$

$x^2 - 2x + \sqrt{3-x} - \sqrt{3-x} - 8 = 0;$

$x^2 - 2x - 8 = 0; \quad 3 - x \geq 0; x \leq 3$

По теореме Виета

(можно через дискриминант):

$x_1 = -2;$

$x_2 = 4$ – не подходит, т.к. $x \leq 3$

Ответ: -2.

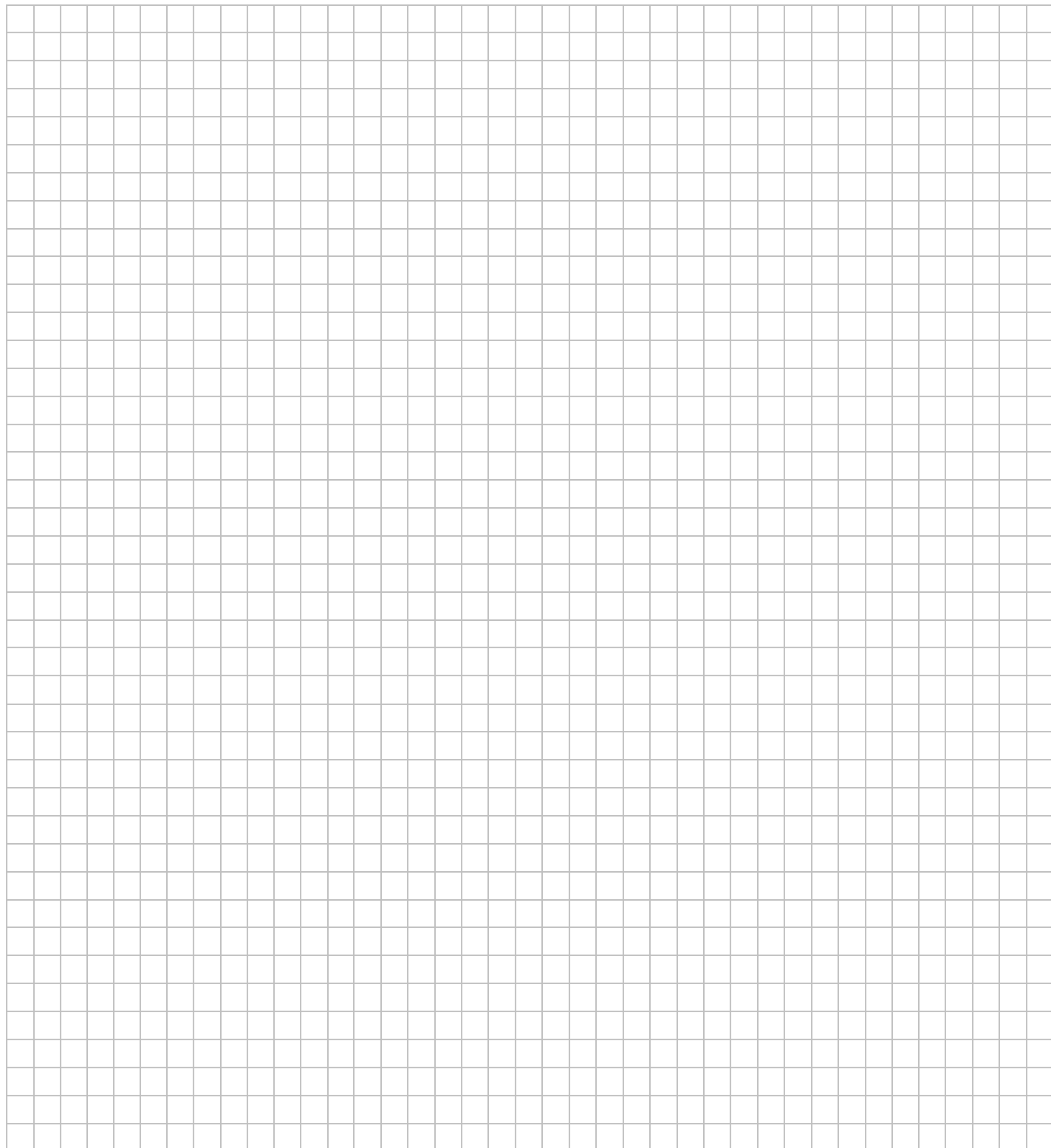
1) $x^2 - 3x + \sqrt{6-x} = \sqrt{6-x} + 28;$

2) $x^2 - 2x + \sqrt{5-x} = \sqrt{5-x} + 24;$

3) $x^2 - 2x + \sqrt{6-x} = \sqrt{6-x} + 35;$

4) $x^2 - 6x + \sqrt{3-x} = \sqrt{3-x} + 7;$

5) $x^2 - 3x + \sqrt{4-x} = \sqrt{4-x} + 40.$



Задание 3. Решите уравнение:

Образец: $(x + 3)^4 - 2(x + 3)^2 - 15 = 0;$

Пусть $(x + 3)^2 = t$, тогда

$t^2 - 2t - 15 = 0;$

По теореме Виета

(можно через дискриминант):

$t_1 = -3; \quad t_2 = 5$

Обратная замена:

$(x + 3)^2 = 5 \quad \text{или} \quad (x + 3)^2 = -3$

$x^2 + 6x + 9 - 5 = 0 \quad \text{решений нет,}$

$x^2 + 6x + 4 = 0 \quad \text{т.к. } (x + 3)^2 \geq 0$

$D = 36 - 4 \cdot 1 \cdot 4 = 20,$

$\sqrt{D} = 2\sqrt{5}$

$x_1 = \frac{-6 + 2\sqrt{5}}{2}; \quad x_2 = \frac{-6 - 2\sqrt{5}}{2}$

$x_1 = -3 + \sqrt{5}; \quad x_2 = -3 - \sqrt{5}$

Ответ: $-3 + \sqrt{5}; -3 - \sqrt{5}.$

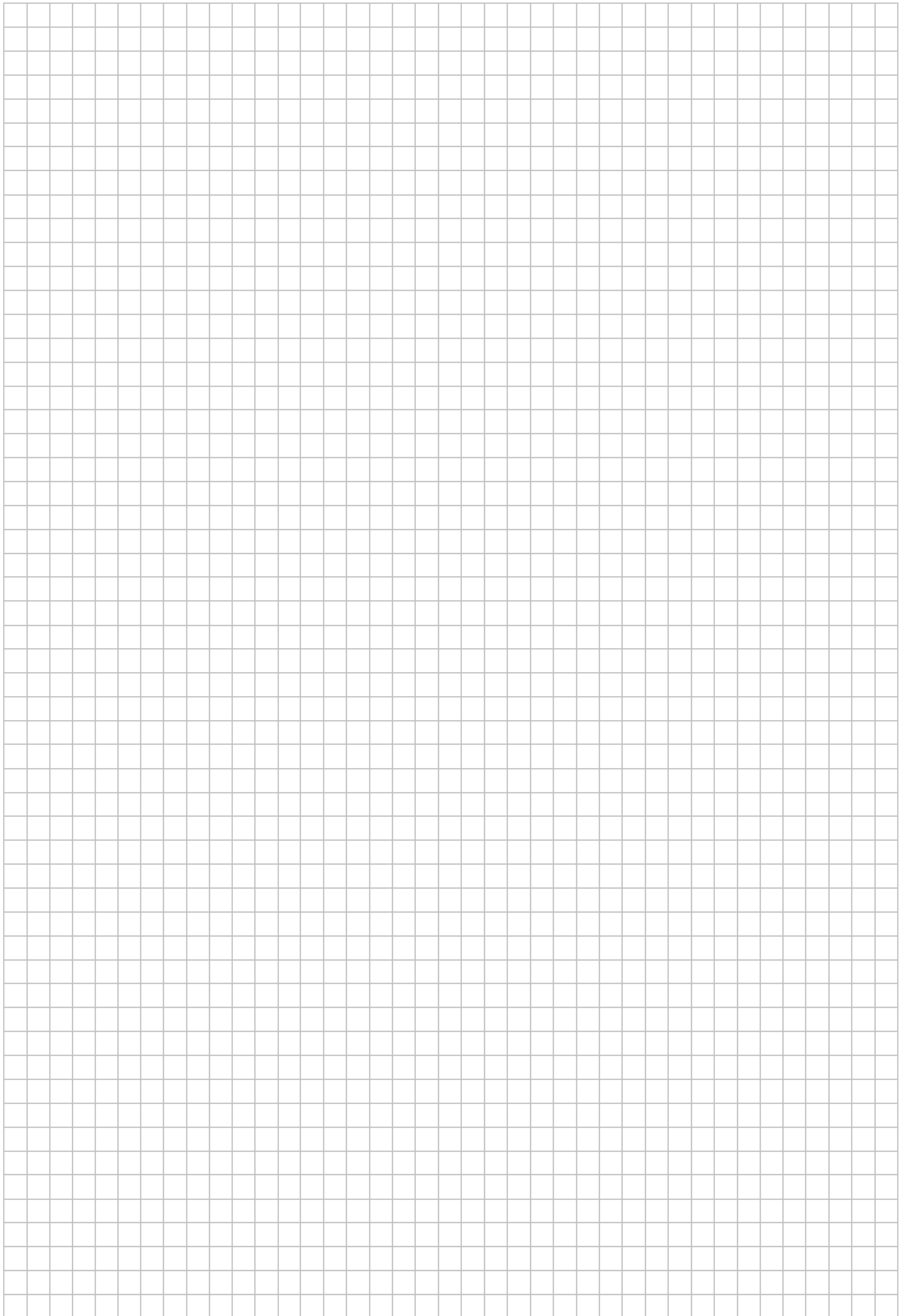
1) $(x - 2)^4 + 3(x - 2)^2 - 10 = 0;$

2) $(x + 2)^4 + 3(x + 2)^2 - 12 = 0;$

3) $(x - 1)^4 - 2(x - 1)^2 - 3 = 0;$

4) $(x + 3)^4 + 2(x + 3)^2 - 8 = 0;$

5) $(x + 4)^4 - 6(x + 4)^2 - 7 = 0.$



Задание 4. Решите уравнение:

Образец: $(x - 4)(x^2 + 2x + 1) = 6(x + 1);$

$(x - 4)(x + 1)^2 - 6(x + 1) = 0;$

$(x + 1)((x - 4)(x + 1) - 6) = 0;$

$x + 1 = 0$ или $x^2 + x - 4x - 4 - 6 = 0$

$x = -1$ $x^2 - 3x - 10 = 0$

По теореме Виета:

$x_1 = 5, x_2 = -2$

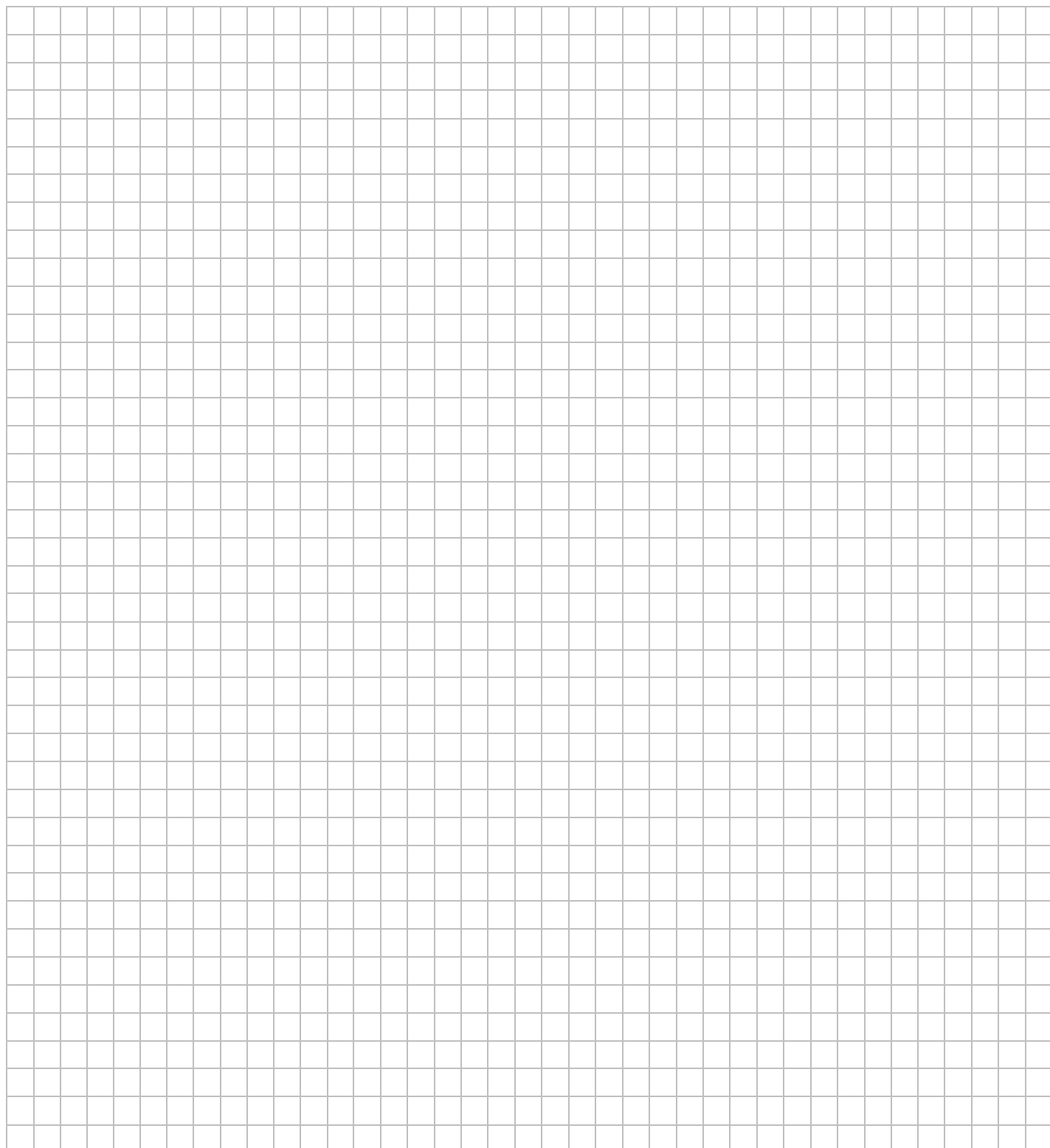
Ответ: -2; -1; 5.

1) $(x - 1)(x^2 + 6x + 9) = 5(x + 3);$

2) $(x - 3)(x^2 + 10x + 25) = 9(x + 5);$

3) $x(x^2 + 8x + 16) = 12(x + 4);$

4) $x(x^2 + 4x + 4) = 8(x + 2).$



Задание 5. Решите уравнение:

Образец: $(x^2 - 49)^2 + (x^2 + 4x - 21)^2 = 0;$

$$x^2 + 4x - 21 = 0$$

По теореме Виета: $x_1 = -7, x_2 = 3$

$$(x - 7)^2(x + 7)^2 + (x + 7)^2(x - 3)^2 = 0;$$

$$(x + 7)^2((x - 7)^2 + (x - 3)^2) = 0;$$

$$(x + 7)^2 = 0 \text{ или } x^2 - 14x + 49 + x^2 - 6x + 9 = 0;$$

$$x = -7 \quad 2x^2 - 20x + 58 = 0 \quad /:2$$

$$x^2 - 10x + 26 = 0$$

$$D = 100 - 4 \cdot 1 \cdot 26 = -4$$

корней нет

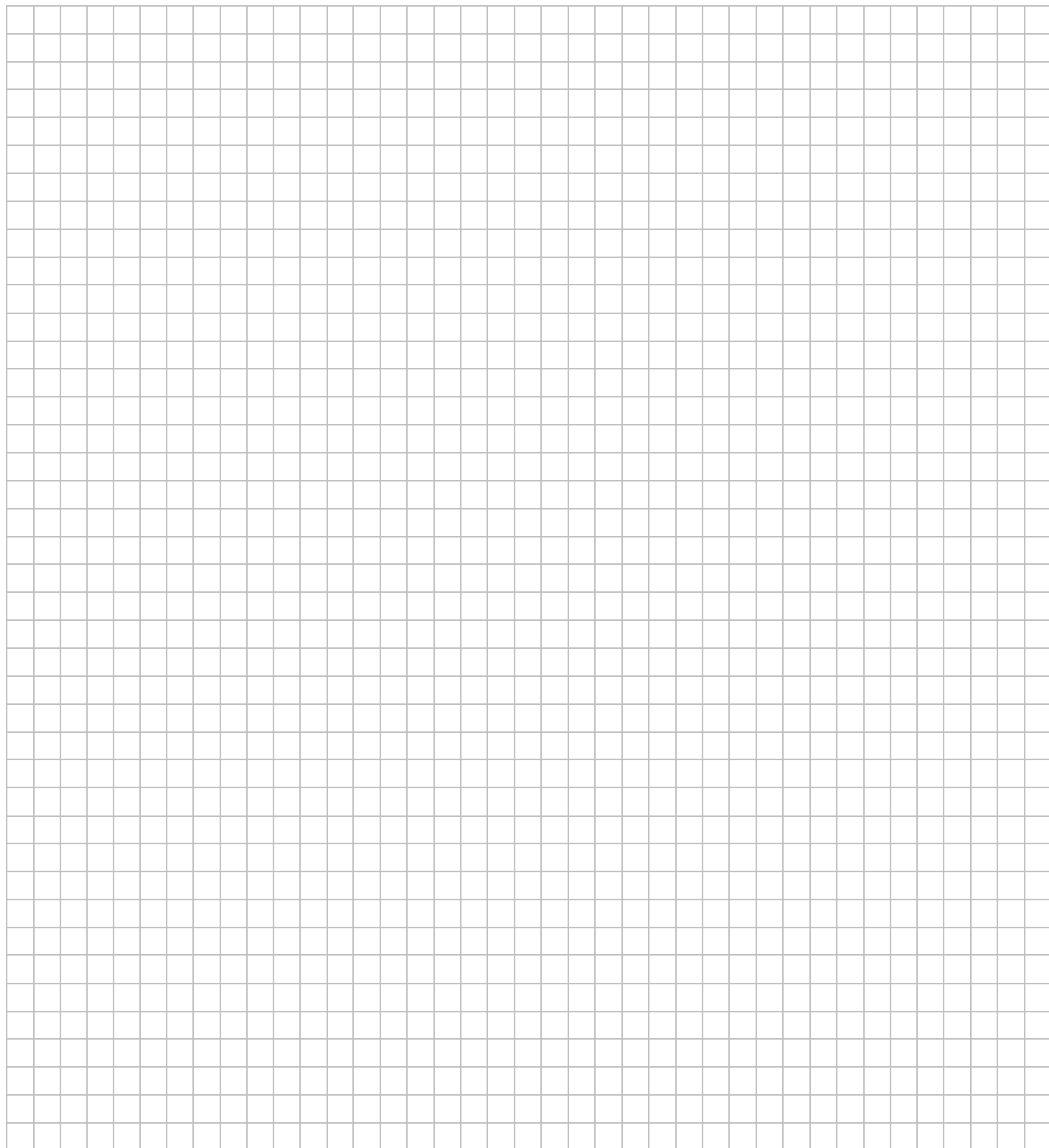
Ответ: -7 .

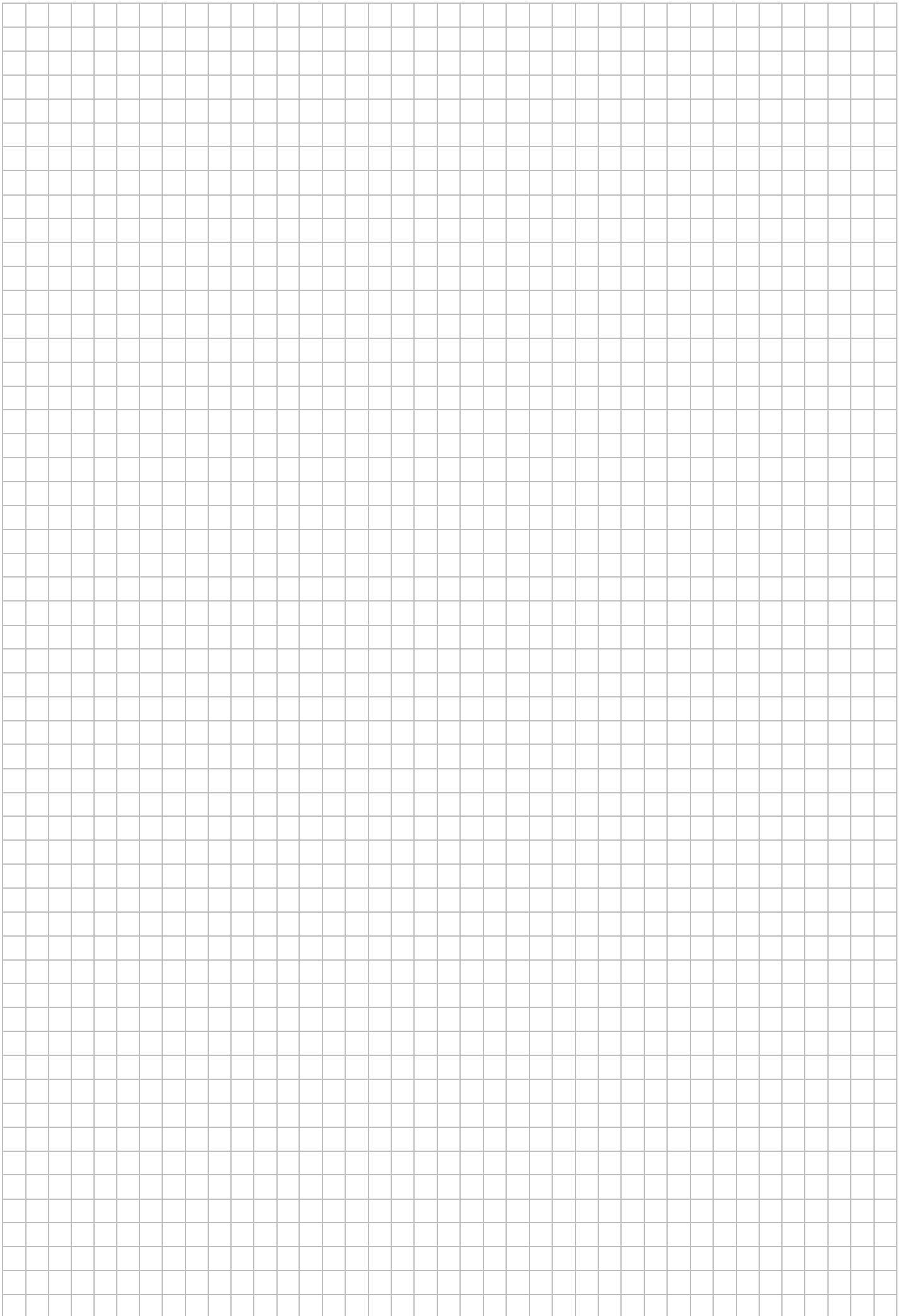
1) $(x^2 - 9)^2 + (x^2 + x - 6)^2 = 0;$

2) $(x^2 - 36)^2 + (x^2 + 4x - 12)^2 = 0;$

3) $(x^2 - 4)^2 + (x^2 - 6x - 16)^2 = 0;$

4) $(x^2 - 1)^2 + (x^2 - 6x - 7)^2 = 0.$





Задание 6. Решите уравнение:

Образец: $x^4 = (2x - 8)^2$;

$x^4 - (2x - 8)^2 = 0$;

$(x^2 - (2x - 8))(x^2 + (2x - 8)) = 0$;

$x^2 - 2x + 8 = 0$ или $x^2 + 2x - 8 = 0$

$D = 4 - 4 \cdot 1 \cdot 8 < 0$ По теореме Виета:

корней нет

$x_1 = -4, \quad x_2 = 2$

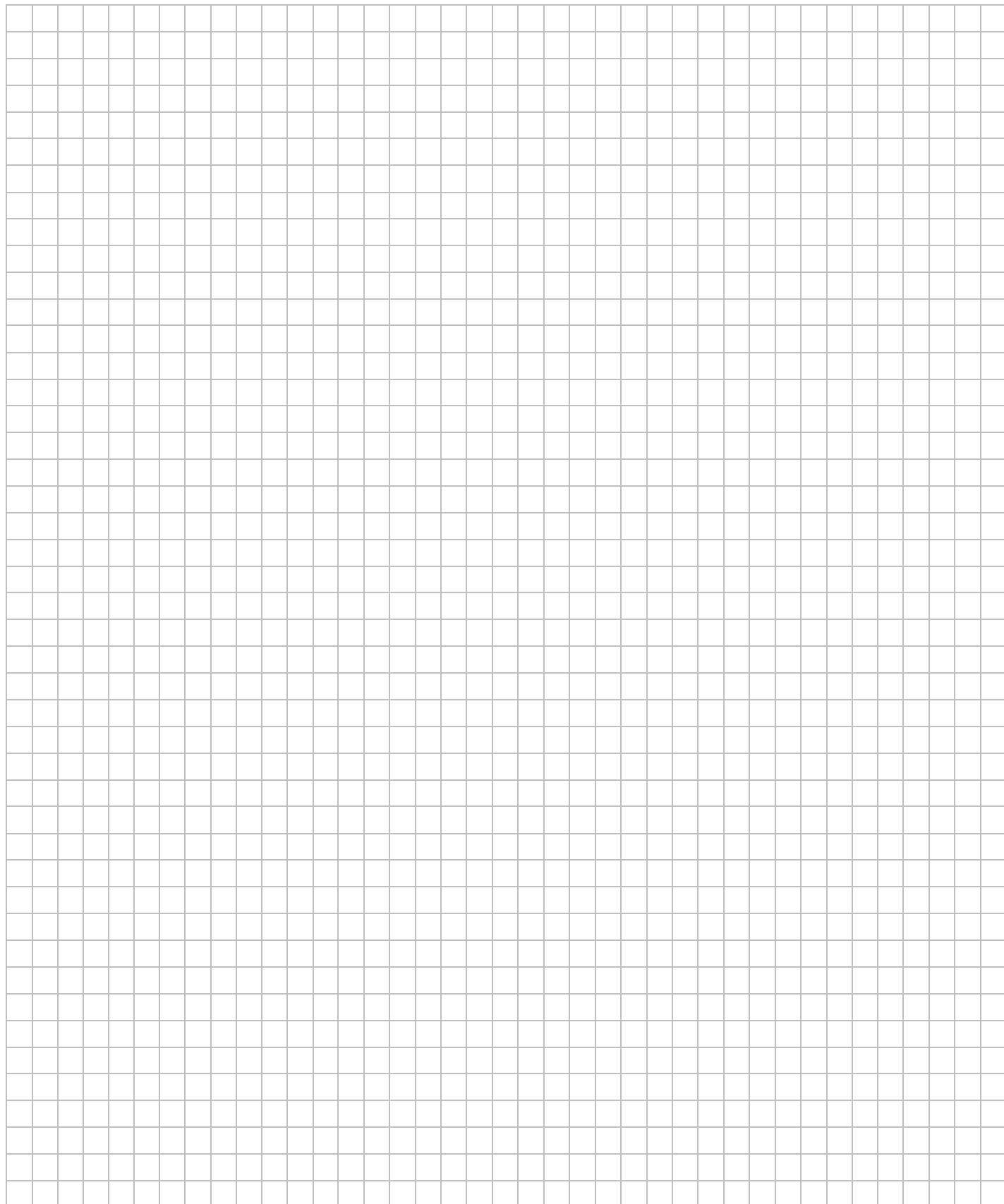
Ответ: $-4; -2$.

1) $x^4 = (2x - 3)^2$;

2) $x^4 = (x - 12)^2$;

3) $x^4 = (4x - 5)^2$;

4) $x^4 = (5x - 14)^2$.



Задание 7. Решите уравнение:

Образец: $\frac{1}{(x-2)^2} - \frac{1}{x-2} - 6 = 0; \quad x-2 \neq 0; \quad x \neq 2$

Пусть $x-2 = t$, тогда

$$\frac{1}{t^2} - \frac{1}{t} - 6 = 0 \quad / \cdot t^2 \neq 0$$

$$1 - t - 6t^2 = 0 \quad / \cdot (-1)$$

$$6t^2 + t - 1 = 0$$

$$D = 1 - 4 \cdot 6 \cdot (-1) = 25, \quad \sqrt{D} = 5$$

$$x_1 = \frac{-1+5}{12}; \quad x_2 = \frac{-1-5}{12}$$

$$x_1 = \frac{1}{3}; \quad x_2 = -0,5$$

Ответ: $-0,5; \frac{1}{3}$.

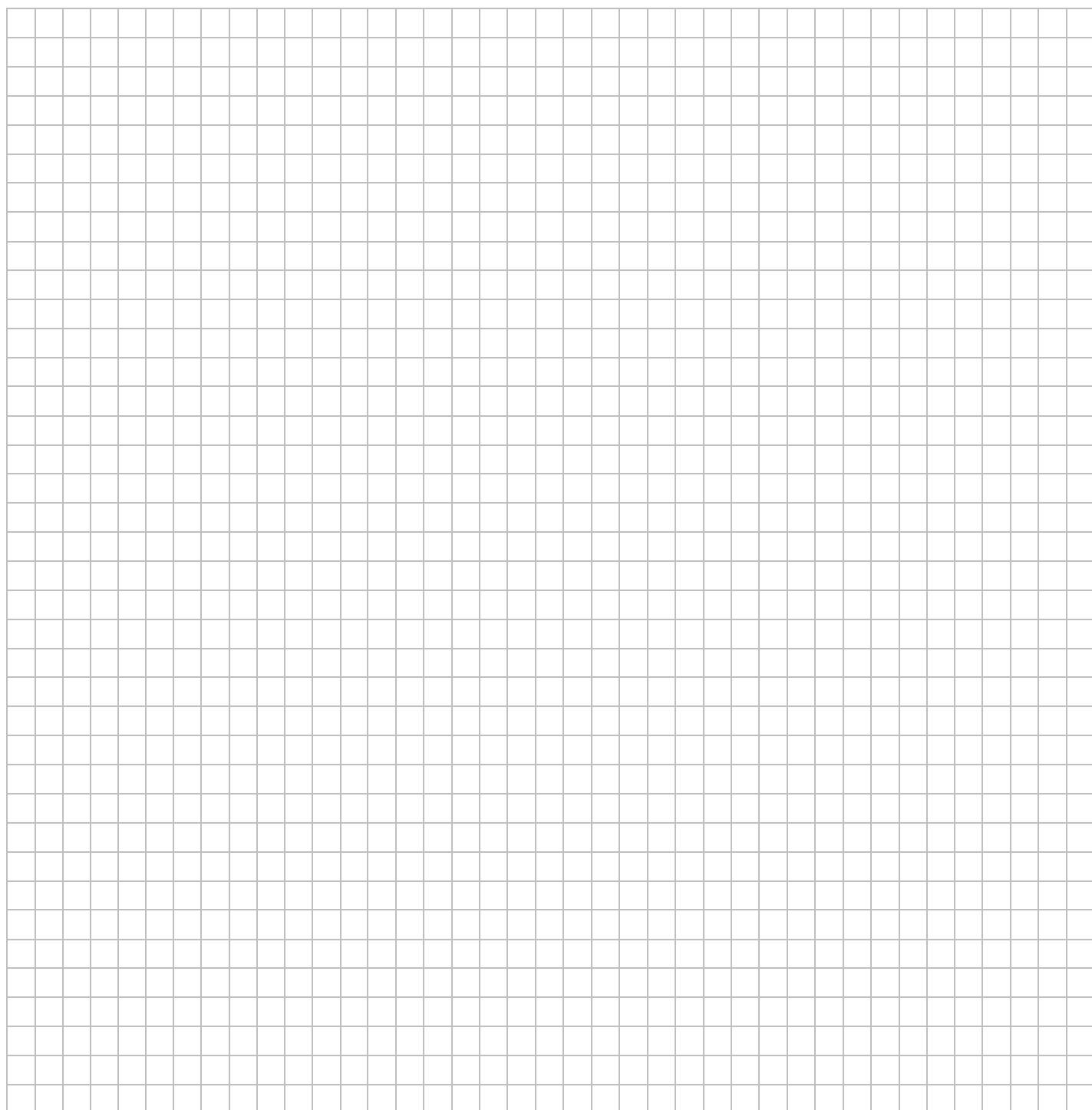
1) $\frac{1}{(x-1)^2} + \frac{4}{x-1} - 12 = 0;$

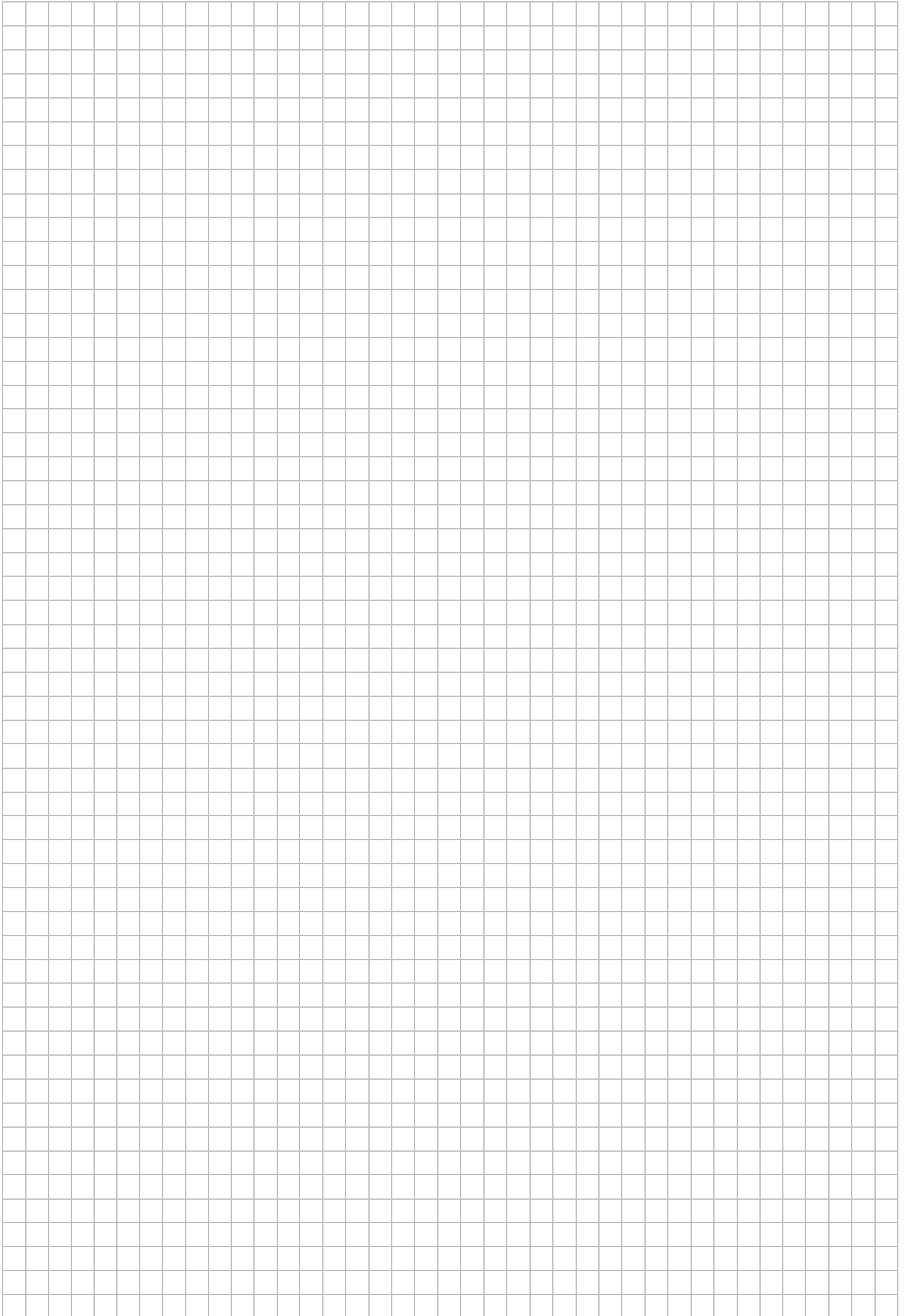
2) $\frac{1}{(x-3)^2} - \frac{6}{x-3} - 16 = 0;$

3) $\frac{1}{(x-4)^2} + \frac{1}{x-4} - 20 = 0;$

4) $\frac{1}{x^2} - \frac{5}{x} - 14 = 0;$

5) $\frac{1}{x^2} - \frac{3}{x} - 4 = 0.$





Задание 8. Решите систему уравнений:

$$\text{Образец: } \begin{cases} 4x^2 - 5x = y, \\ 8x - 10 = y; \end{cases} \Rightarrow \begin{cases} 4x^2 - 5x = 8x - 10; \\ 8x - 10 = y; \end{cases} \Rightarrow \begin{cases} 4x^2 - 13x + 10 = 0, \\ y = 8x - 10; \end{cases}$$

$$4x^2 - 13x + 10 = 0$$

$$D = 169 - 4 \cdot 4 \cdot 10 = 9, \sqrt{D} = 3$$

$$x_1 = \frac{13 + 3}{8}; \quad x_2 = \frac{13 - 3}{8}$$

$$x_1 = 2; \quad x_2 = 1,25$$

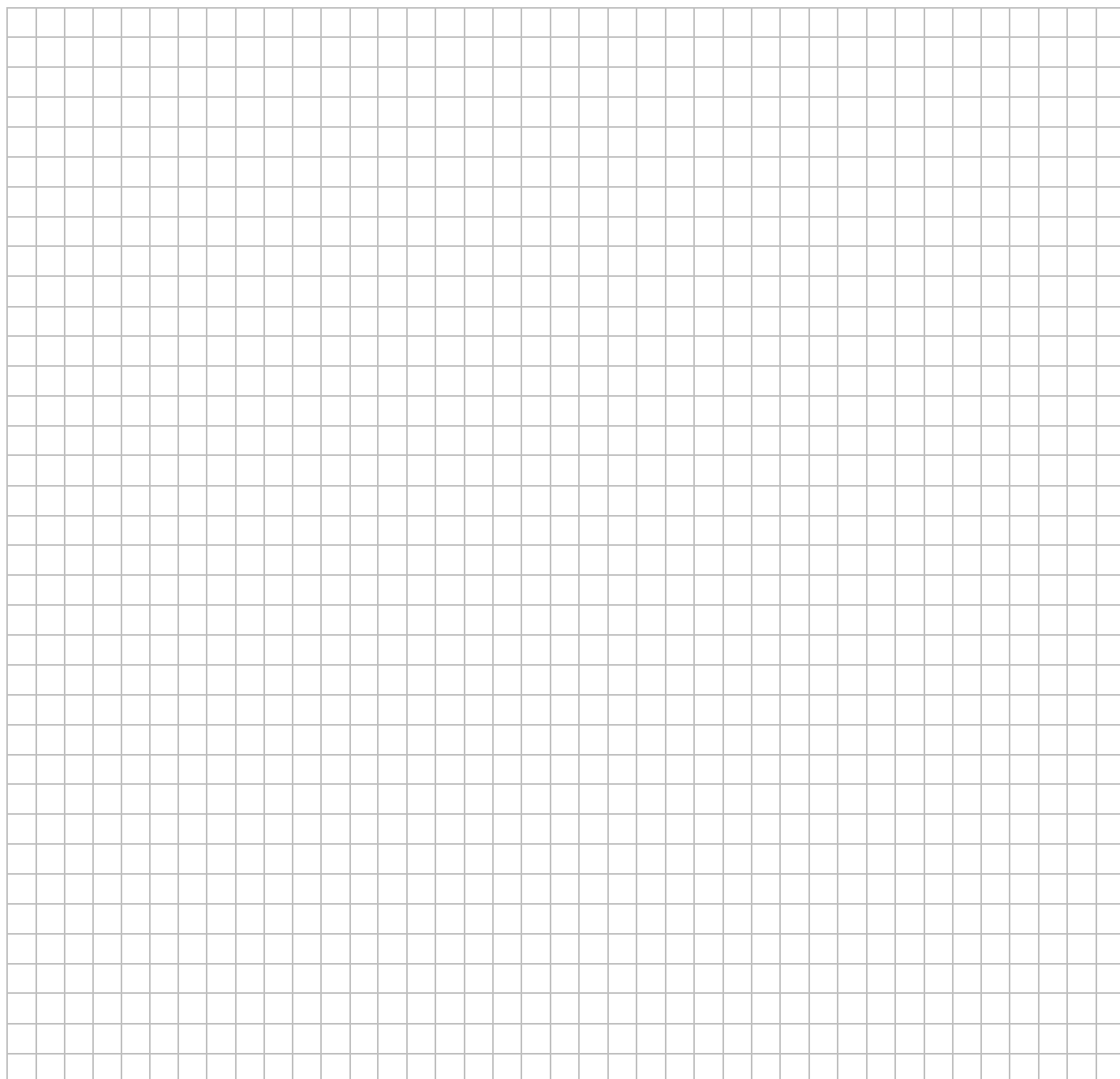
$$\begin{cases} x = 2, \\ x = 1,25, \\ y = 8x - 10; \end{cases} \Rightarrow \begin{cases} x = 2, \\ y = 8 \cdot 2 - 10, \\ x = 1,25, \\ y = 8 \cdot 1,25 - 10, \end{cases} \Rightarrow \begin{cases} x = 2, \\ y = 6, \\ x = 1,25, \\ y = 0. \end{cases}$$

Ответ: (2; 6), (1,25; 0).

1) $\begin{cases} 5x^2 - 9x = y, \\ 5x - 9 = y; \end{cases}$

2) $\begin{cases} 2x^2 - x = y, \\ 2x - 1 = y; \end{cases}$

3) $\begin{cases} 4x^2 - 3x = y, \\ 8x - 6 = y. \end{cases}$



Задание 9. Решите систему уравнений:

Образец:

$$\begin{cases} 5x^2 + y = 12, \\ 9x^2 - y = 2; \end{cases} \Rightarrow \begin{cases} 14x^2 = 14; \\ 9x^2 - y = 2; \end{cases} \Rightarrow \begin{cases} x^2 = 1, \\ y = 9x^2 - 2; \end{cases} \Rightarrow \begin{cases} x = 1, \\ x = -1, \\ y = 9x^2 - 2; \end{cases}$$

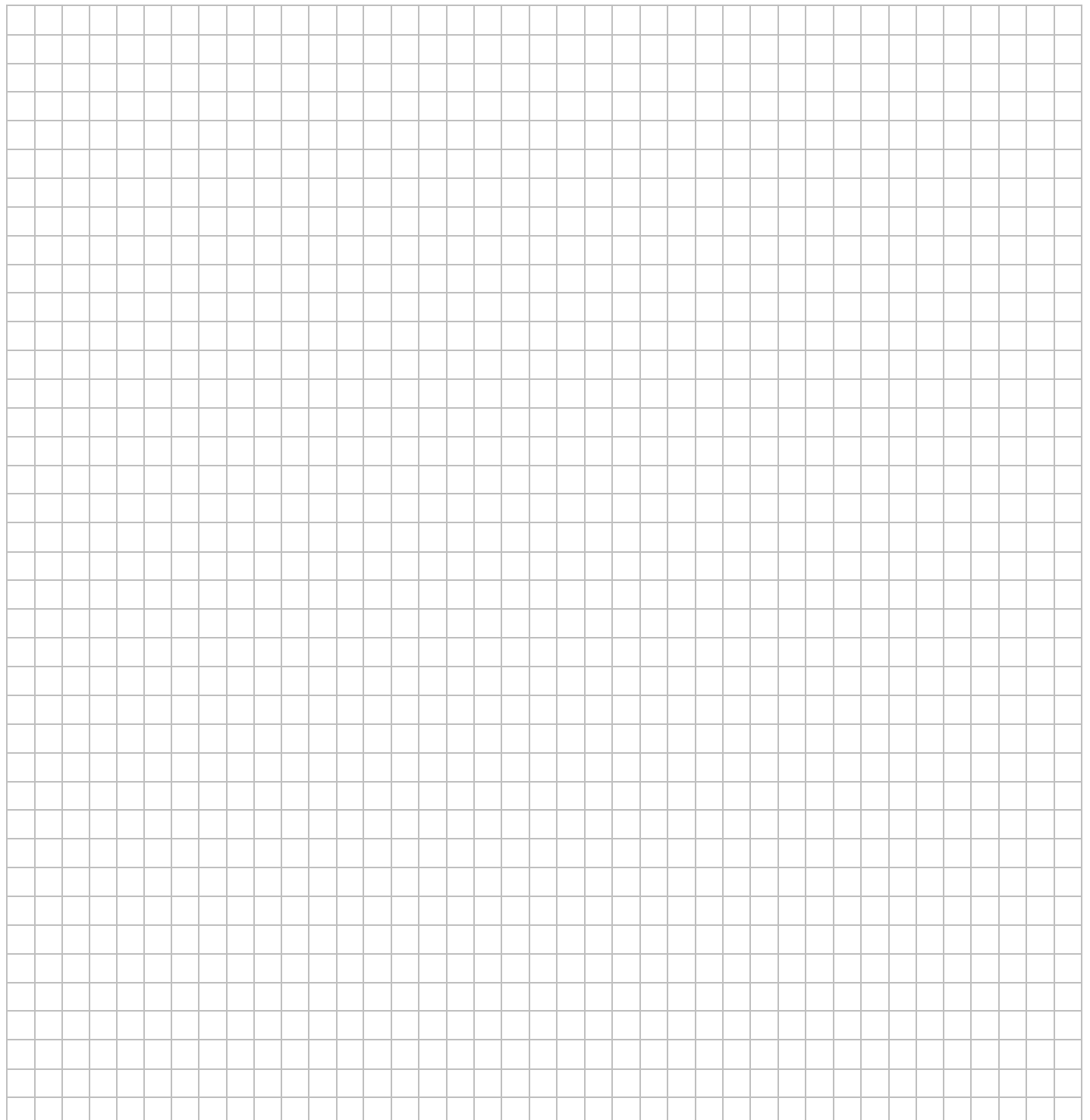
$$\begin{cases} x = 1, \\ y = 9 \cdot 1^2 - 2, \\ x = -1, \\ y = 9 \cdot (-1)^2 - 2, \end{cases} \Rightarrow \begin{cases} x = 1, \\ y = 7, \\ x = -1, \\ y = 7. \end{cases}$$

Ответ: (1; 7), (-1; 7).

1)
$$\begin{cases} 2x^2 + y = 9, \\ 3x^2 - y = 11; \end{cases}$$

2)
$$\begin{cases} x^2 + y = 5, \\ 6x^2 - y = 2; \end{cases}$$

3)
$$\begin{cases} 6x^2 + y = 14, \\ 12x^2 - y = 4. \end{cases}$$



Задание 10. Решите систему уравнений:

$$\text{Образец: } \begin{cases} 5x^2 + y^2 = 61, & / \cdot 3 \\ 15x^2 + 3y^2 = 61x; \end{cases} \Rightarrow \begin{cases} 15x^2 + 3y^2 = 183, \\ 15x^2 + 3y^2 = 61x; \end{cases} \Rightarrow \begin{cases} 61x = 183, \\ 5x^2 + y^2 = 61; \end{cases}$$

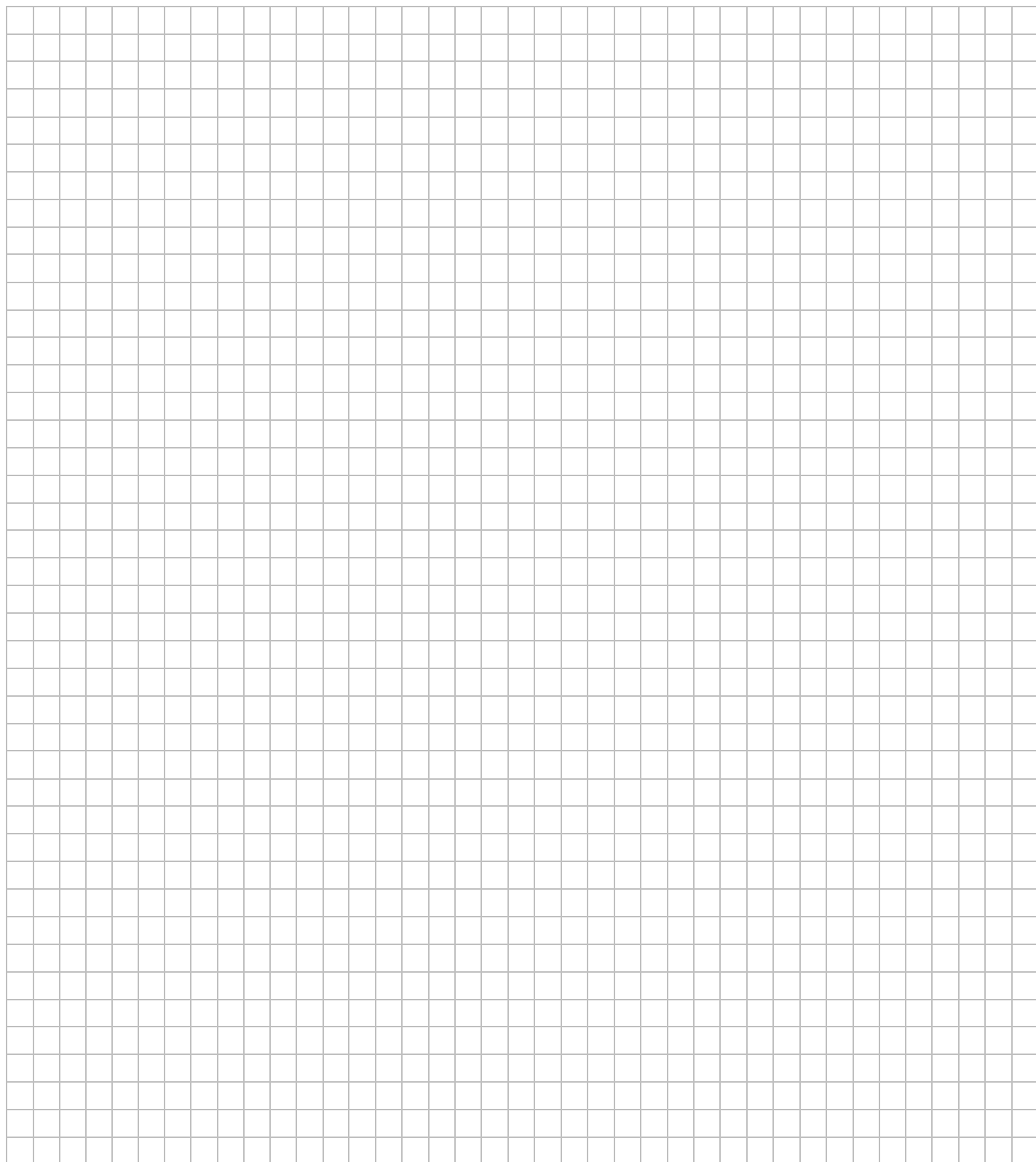
$$\begin{cases} x = 3, \\ 5 \cdot 3^2 + y^2 = 61; \end{cases} \Rightarrow \begin{cases} x = 3, \\ y^2 = 16; \end{cases} \Rightarrow \begin{cases} x = 3, \\ y = 4, \\ y = -4; \end{cases}$$

Ответ: (3; 4), (3; -4).

1)
$$\begin{cases} 2x^2 + 3y^2 = 11, \\ 4x^2 + 6y^2 = 11x; \end{cases}$$

2)
$$\begin{cases} 5x^2 + y^2 = 36, \\ 10x^2 + 2y^2 = 36x; \end{cases}$$

3)
$$\begin{cases} x^2 + 3y^2 = 31, \\ 2x^2 + 6y^2 = 31x. \end{cases}$$



Задание 11. Решите неравенство:

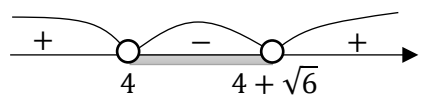
Образец: $(x - 4)^2 < \sqrt{6}(x - 4)$;

$(x - 4)^2 - \sqrt{6}(x - 4) < 0$;

$(x - 4)(x - 4 - \sqrt{6}) < 0$;

$x - 4 = 0$ $x - 4 - \sqrt{6} = 0$

$x = 4$ $x = 4 + \sqrt{6}$

Ответ: $(4; 4 + \sqrt{6})$

1) $(x - 9)^2 < \sqrt{2}(x - 9)$;

2) $(x - 2)^2 < \sqrt{3}(x - 2)$;

3) $(x - 7)^2 < \sqrt{11}(x - 7)$;

4) $(x - 11)^2 < \sqrt{5}(x - 11)$.

Задание 12. Решите неравенство:

Образец:
$$-\frac{12}{x^2 - 7x - 8} \leq 0;$$

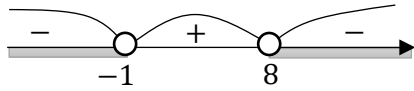
$$x^2 - 7x - 8 = 0;$$

По теореме Виета:

$$x_1 = 8; \quad x_2 = -1$$

$$x^2 - 7x - 8 = (x - 8)(x + 1)$$

$$-\frac{12}{(x - 8)(x + 1)} \leq 0;$$

Ответ: $(-\infty; -1) \cup (8; +\infty)$.

1)
$$-\frac{23}{x^2 + 6x - 16} \leq 0;$$

2)
$$-\frac{10}{x^2 + 5x - 14} \leq 0.$$

Задание 13. Решите неравенство:

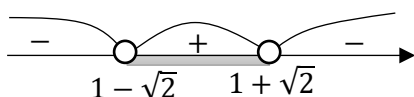
Образец:
$$\frac{-12}{(x - 1)^2 - 2} \geq 0;$$

$$\frac{-12}{(x - 1)^2 - (\sqrt{2})^2} \geq 0;$$

$$\frac{-12}{(x - 1 - \sqrt{2})(x - 1 + \sqrt{2})} \geq 0;$$

$$x - 1 - \sqrt{2} = 0 \quad x - 1 + \sqrt{2} = 0$$

$$x = 1 + \sqrt{2} \quad x = 1 - \sqrt{2}$$

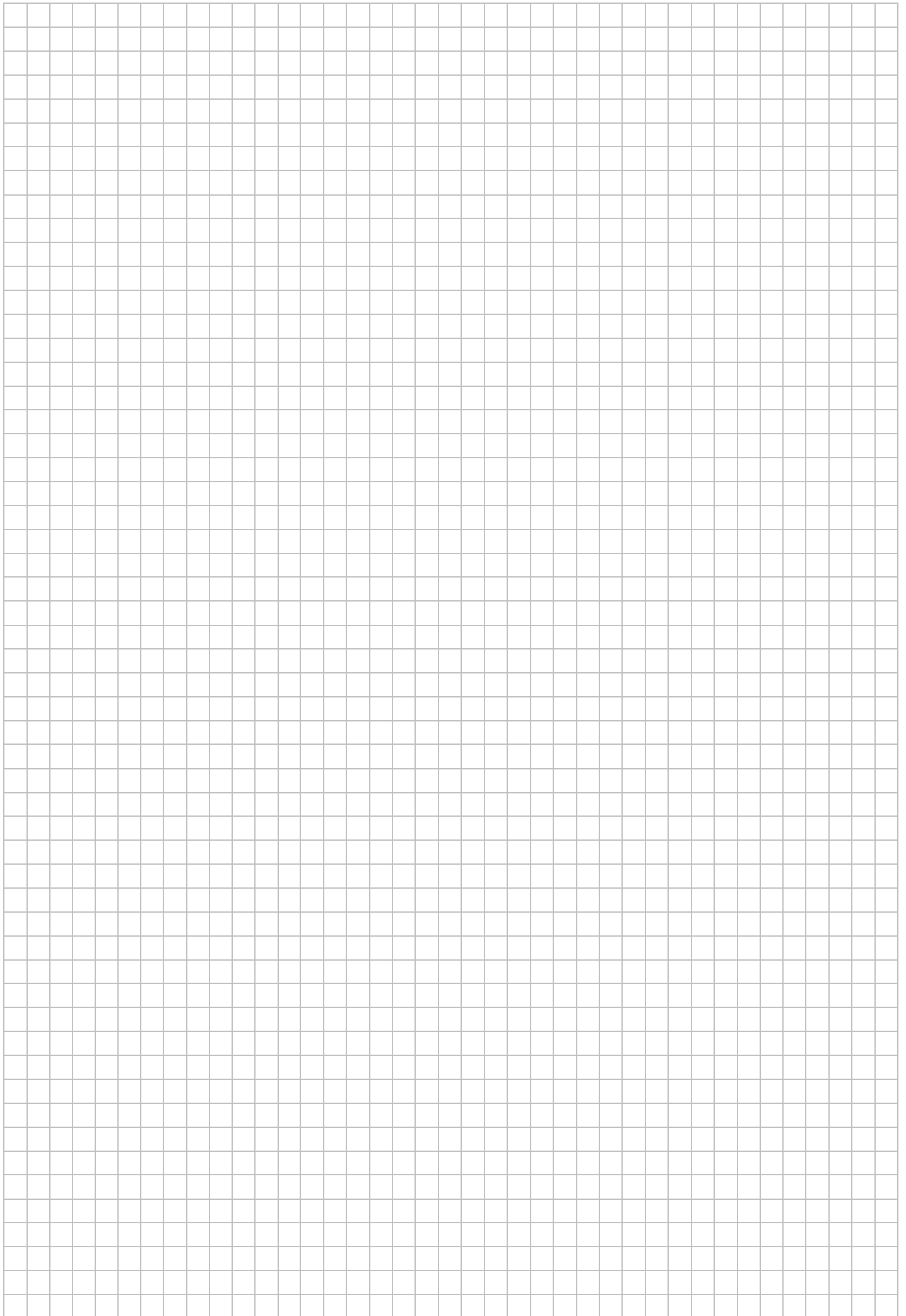
Ответ: $(1 - \sqrt{2}; 1 + \sqrt{2})$

1)
$$\frac{-10}{(x - 3)^2 - 5} \geq 0;$$

2)
$$\frac{-18}{(x + 4)^2 - 10} \geq 0;$$

3)
$$\frac{-19}{(x + 5)^2 - 6} \geq 0;$$

4)
$$\frac{-17}{(x + 3)^2 - 7} \geq 0.$$



Задание 14. Найдите значение выражения при данном условии:Образец: $33a - 23b + 71$, если $\frac{3a - 4b + 8}{4a - 3b + 8} = 9$.

$$\frac{3a - 4b + 8}{4a - 3b + 8} = 9;$$

$$3a - 4b + 8 = 9(4a - 3b + 8);$$

$$3a - 4b + 8 = 36a - 27b + 72;$$

$$36a - 3a - 27b + 4b = 8 - 72;$$

$$33a - 23b = -64.$$

$$33a - 23b + 71 = -64 + 71 = 7.$$

Ответ: 7.

1) $41a - b + 45$, если $\frac{a - 6b + 5}{6a - b + 5} = 7$;

2) $39a - 15b + 25$, если $\frac{3a - 6b + 4}{6a - 3b + 4} = 7$;

3) $61a - 11b + 50$, если $\frac{2a - 7b + 5}{7a - 2b + 5} = 9$;

4) $19a - 7b + 12$, если $\frac{5a - 8b + 2}{8a - 5b + 2} = 3$.



Домашнее задание.

1) $x^3 + 2x^2 - 9x - 18 = 0;$

2) $x^3 + 3x^2 - x - 3 = 0;$

3) $x^3 + 6x^2 = 9x + 54;$

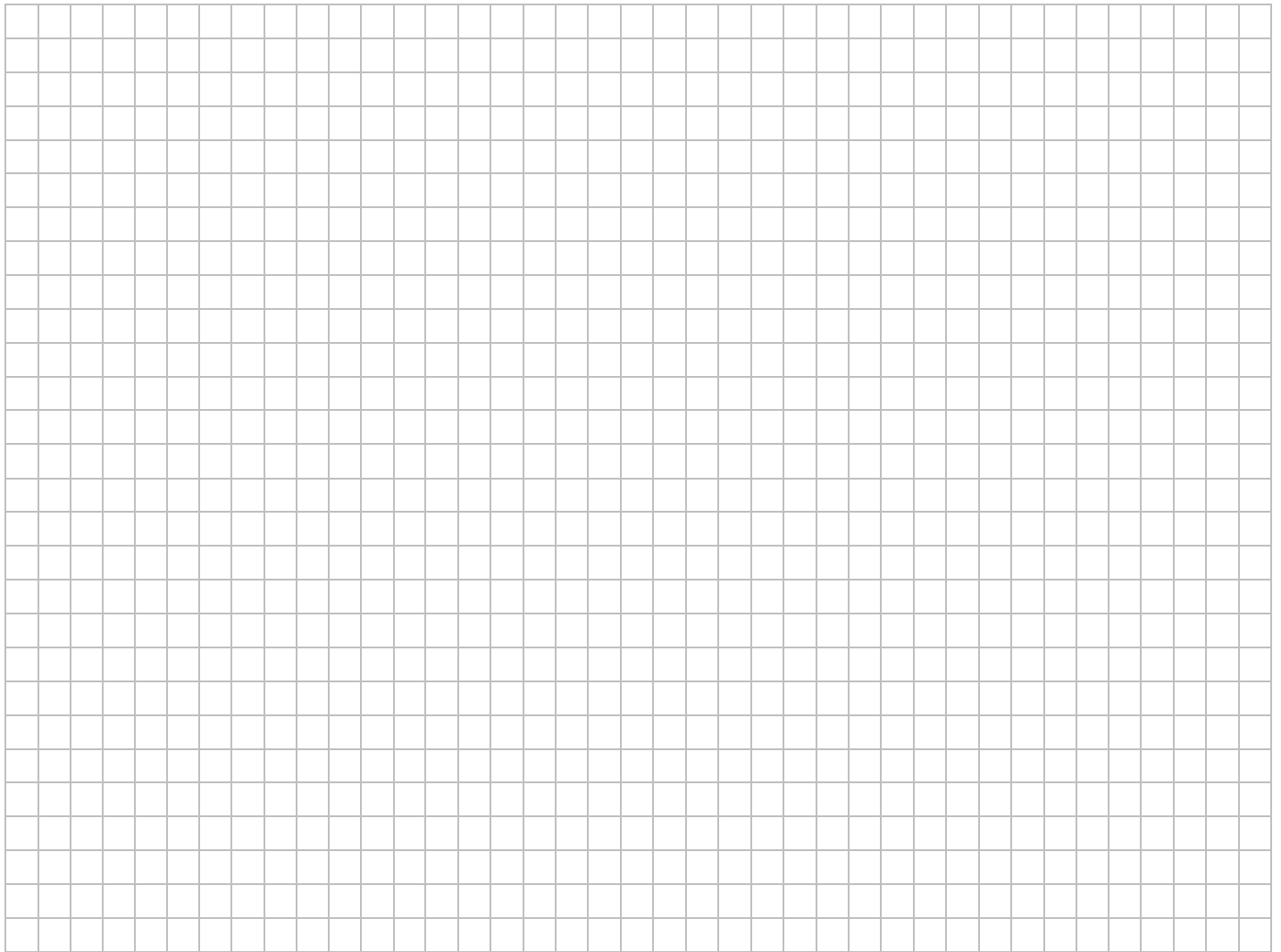
4) $x^3 + 3x^2 = 16x + 48;$

5) $x^2 - 2x + \sqrt{4-x} = \sqrt{4-x} + 15;$

6) $x^2 - 3x + \sqrt{5-x} = \sqrt{5-x} + 18;$

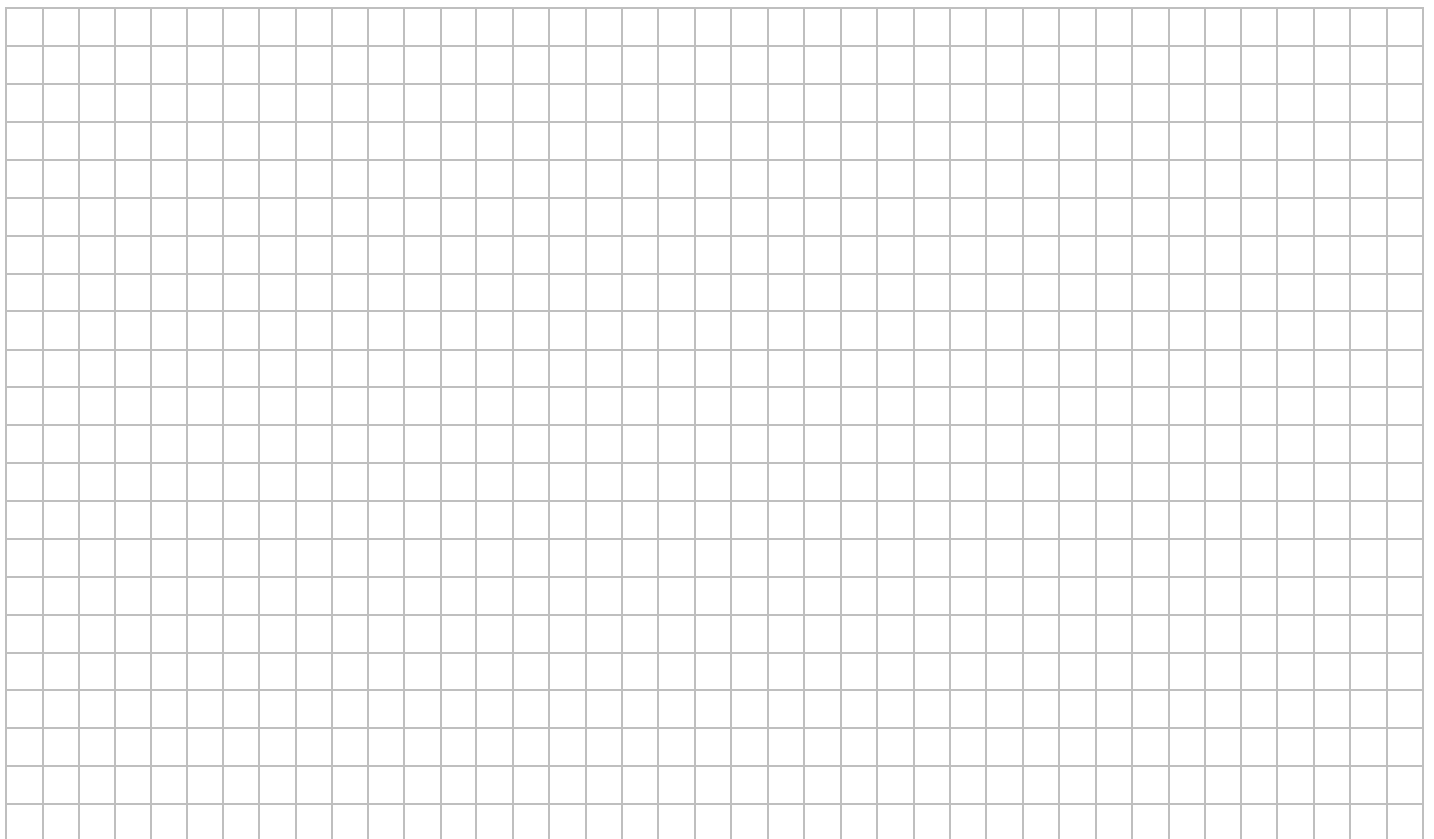
7) $(x + 2)^4 - 4(x + 2)^2 - 5 = 0;$

8) $(x + 1)^4 + (x + 1)^2 - 6 = 0;$



9) $x(x^2 + 2x + 1) = 2(x + 1);$

10) $x(x^2 + 4x + 4) = 3(x + 2);$



11) $(x - 1)(x^2 + 8x + 16) = 6(x + 4);$

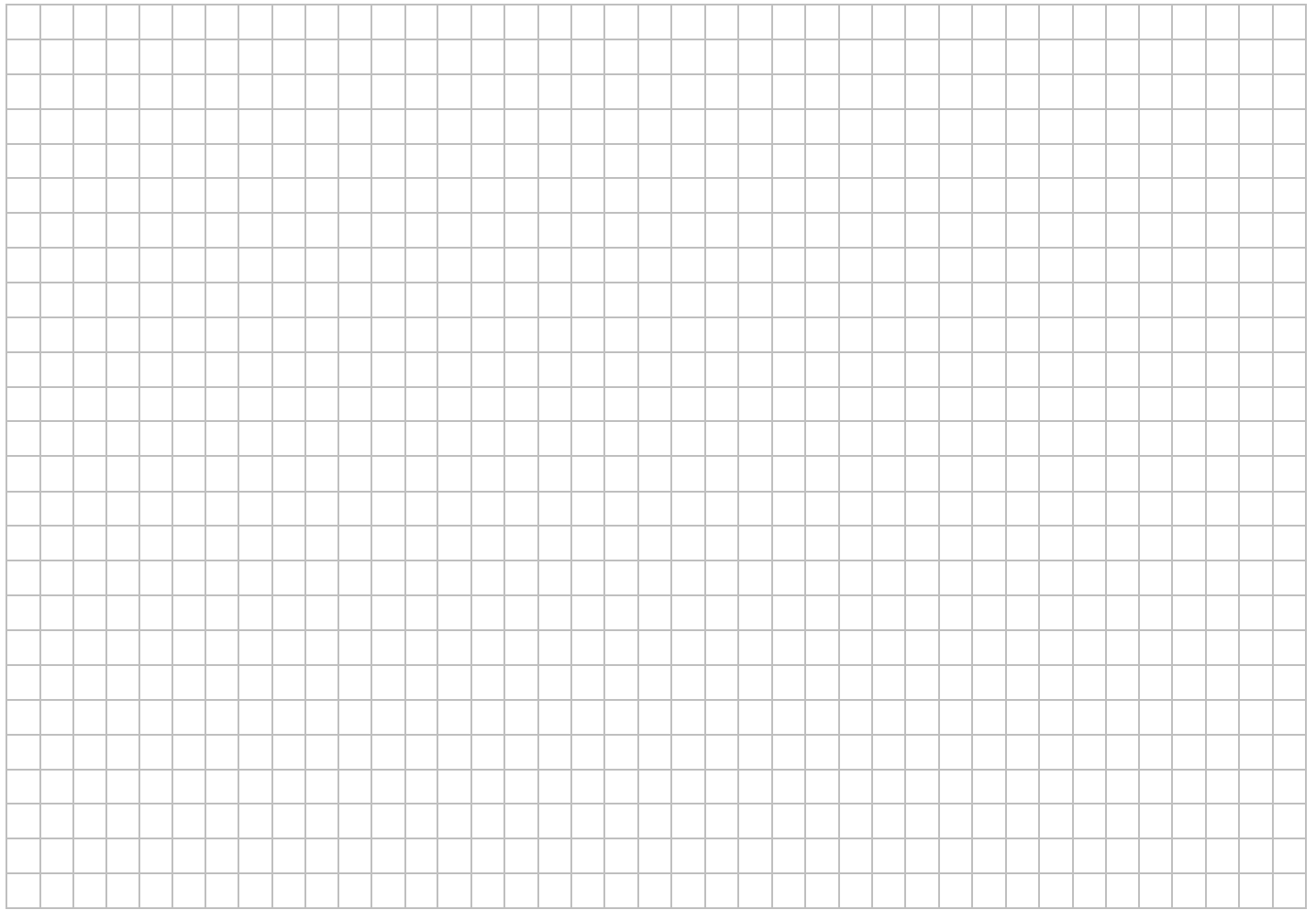
12) $(x - 2)(x^2 + 6x + 9) = 6(x + 3);$

13) $(x^2 - 4)^2 + (x^2 - 3x - 10)^2 = 0;$

14) $(x^2 - 16)^2 + (x^2 + x - 12)^2 = 0;$

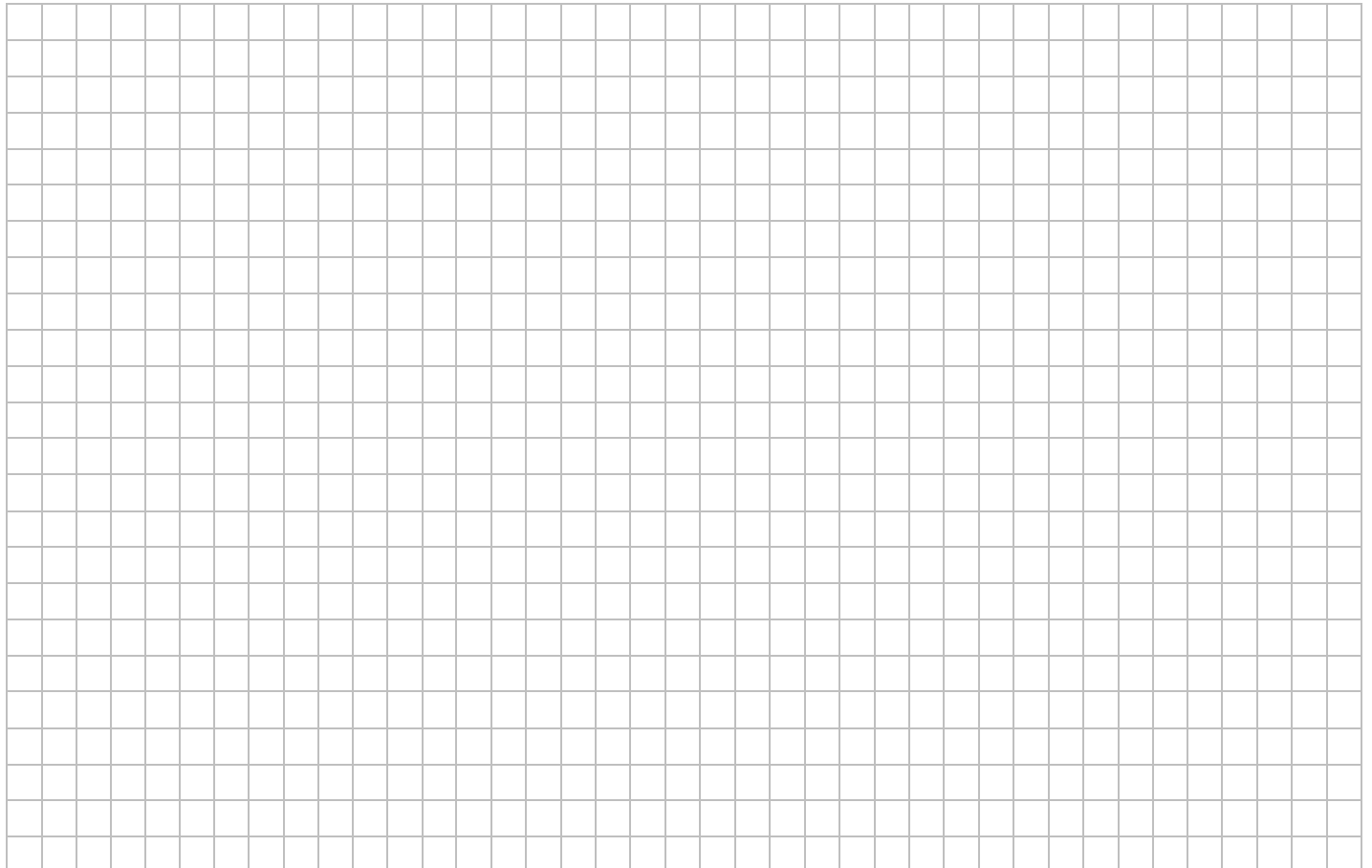
15) $(x^2 - 9)^2 + (x^2 - 2x - 15)^2 = 0;$

16) $(x^2 - 25)^2 + (x^2 + 2x - 15)^2 = 0;$



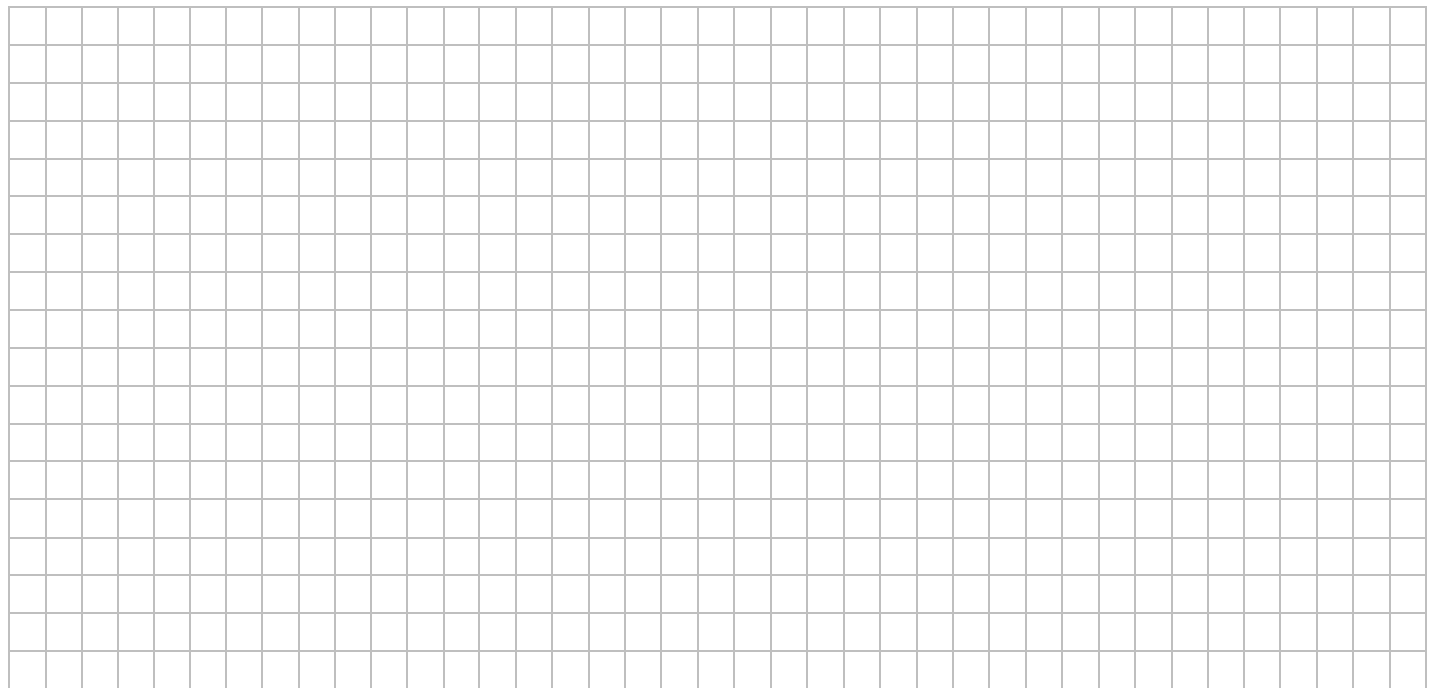
17) $x^4 = (x - 20)^2;$

18) $x^4 = (x - 6)^2;$



19) $x^4 = (2x - 15)^2;$

20) $x^4 = (4x - 5)^2;$



21) $\frac{1}{(x-3)^2} + \frac{3}{x-3} - 4 = 0;$

22) $\frac{1}{(x-1)^2} + \frac{2}{x-1} - 3 = 0;$



$$23) \frac{1}{x^2} + \frac{1}{x} - 12 = 0;$$

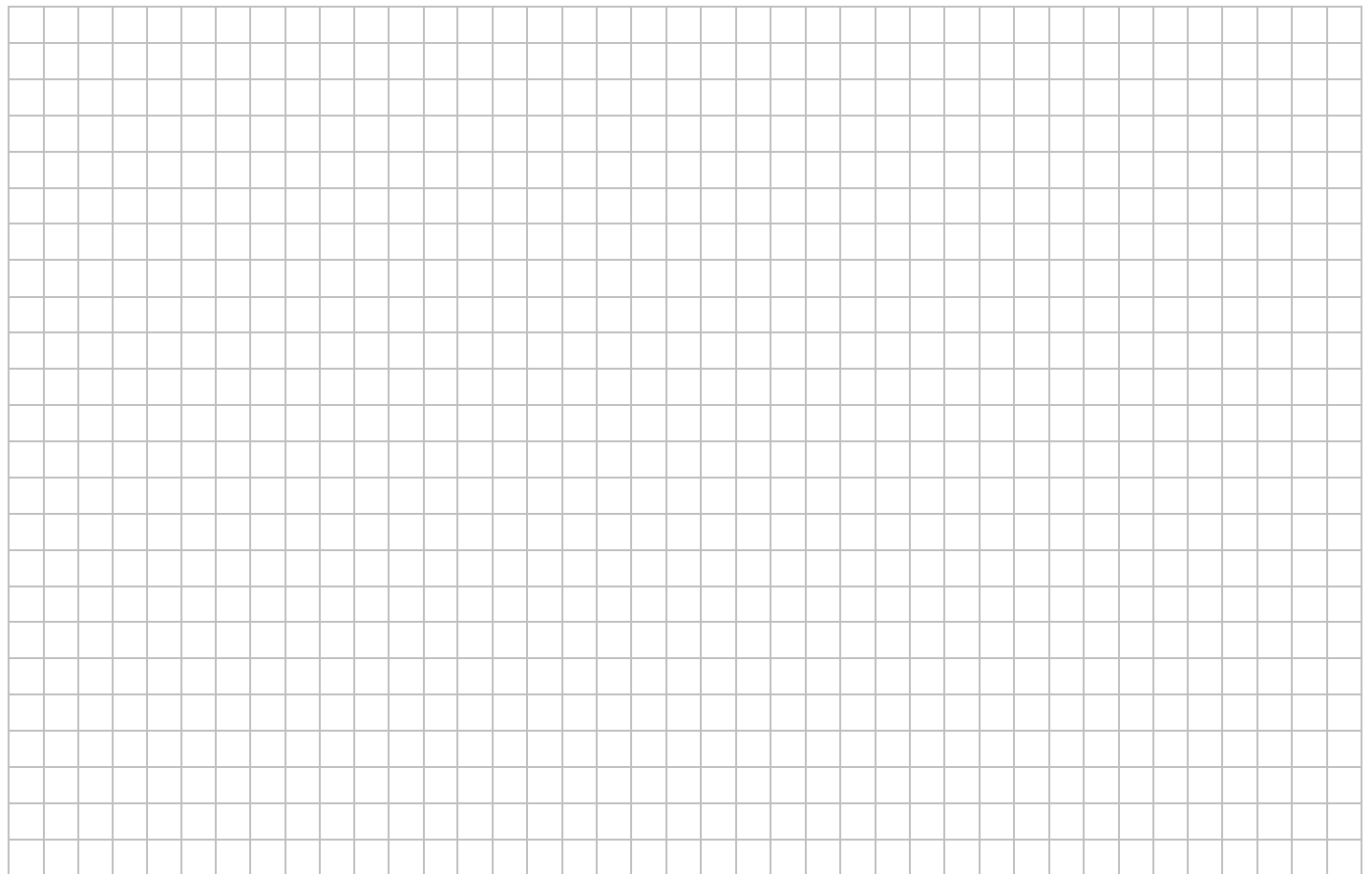
$$24) \frac{1}{x^2} - \frac{6}{x} - 7 = 0;$$

$$25) \begin{cases} 9x^2 - 14x = y, \\ 9x - 14 = y; \end{cases}$$

$$26) \begin{cases} 7x^2 - 5x = y, \\ 7x - 5 = y; \end{cases}$$

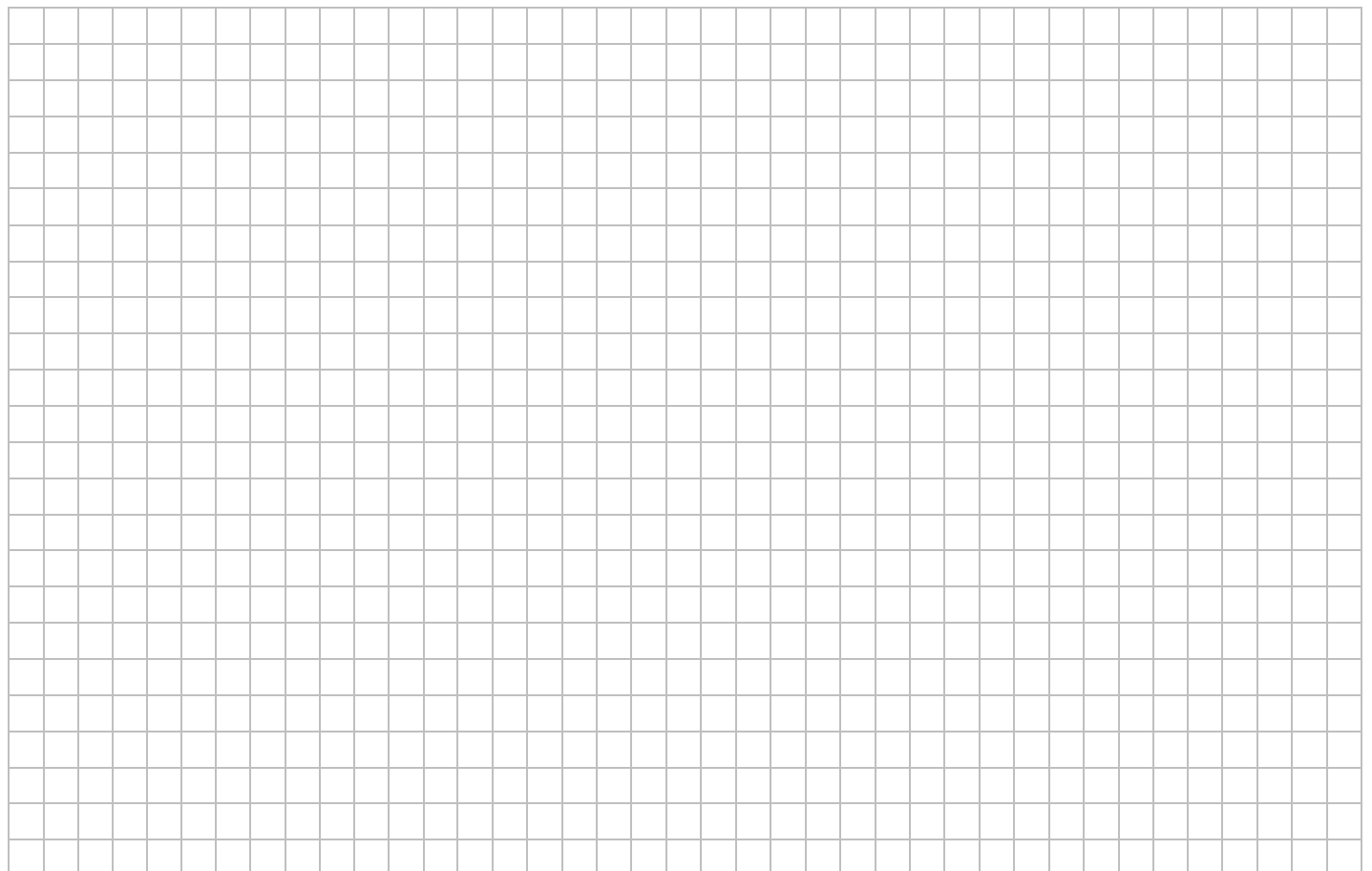
$$27) \begin{cases} 4x^2 + y = 9, \\ 8x^2 - y = 3; \end{cases}$$

$$28) \begin{cases} x^2 + y = 7, \\ 2x^2 - y = 5; \end{cases}$$



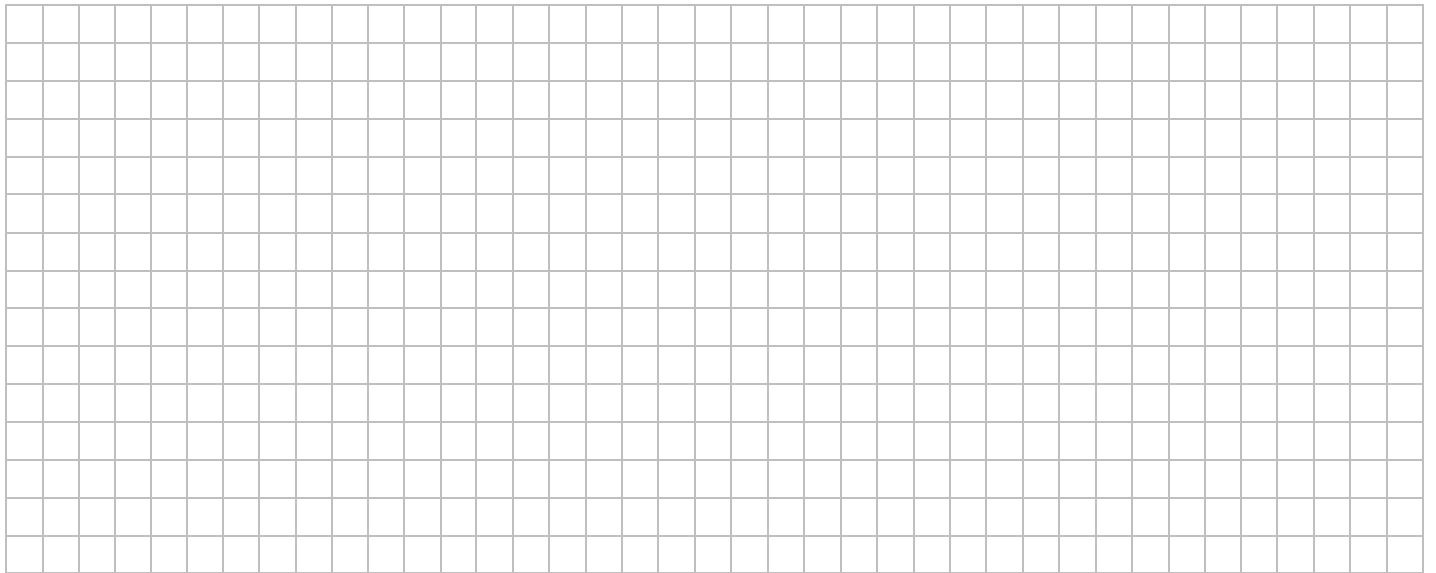
$$29) \begin{cases} 3x^2 + 2y^2 = 50, \\ 12x^2 + 8y^2 = 50x; \end{cases}$$

$$30) \begin{cases} 3x^2 + 2y^2 = 45, \\ 9x^2 + 6y^2 = 45x; \end{cases}$$



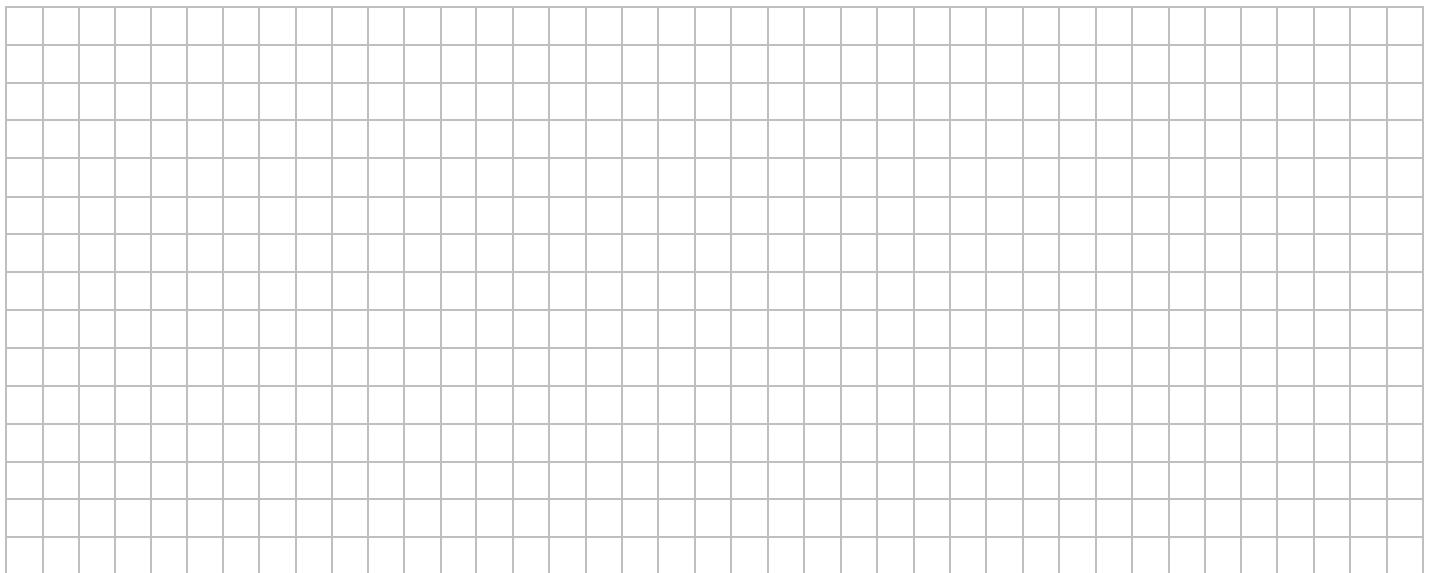
31) $(x - 8)^2 < \sqrt{3}(x - 8);$

32) $(x - 5)^2 < \sqrt{4}(x - 5);$



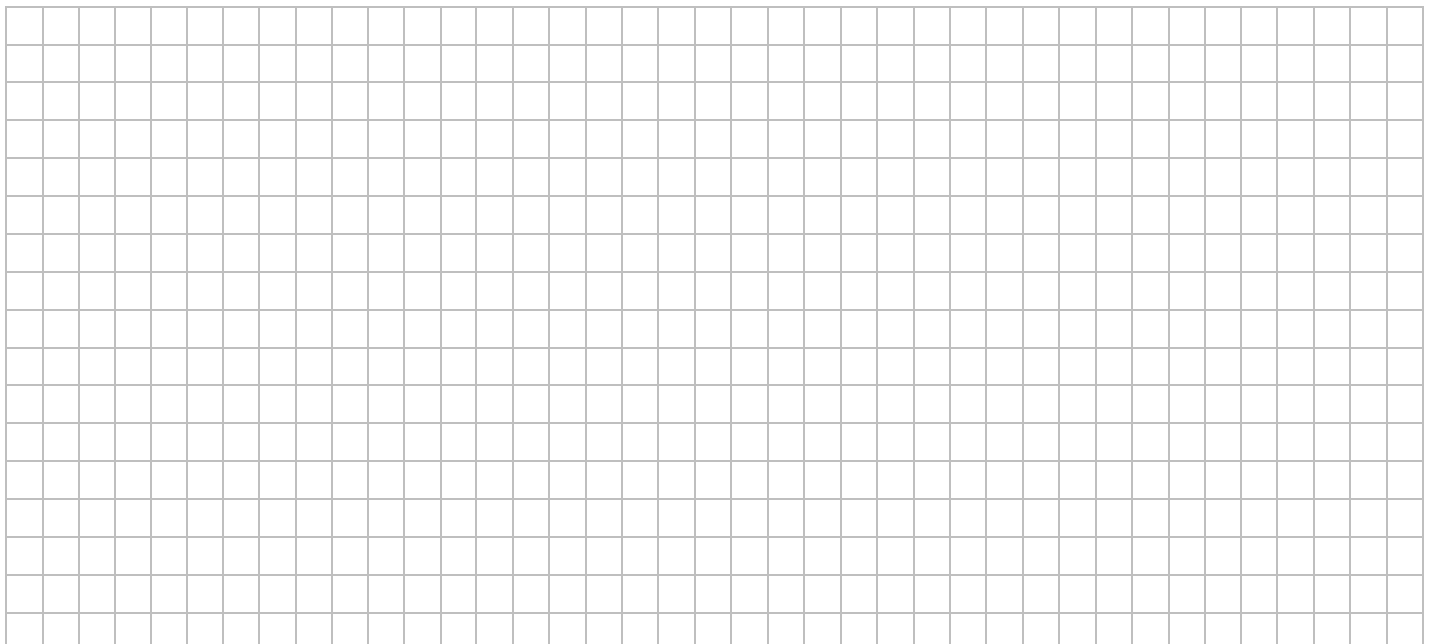
33) $(x - 1)^2 < \sqrt{2}(x - 1);$

34) $(x - 3)^2 < \sqrt{7}(x - 3);$



35) $-\frac{19}{x^2 + x - 12} \leq 0;$

36) $-\frac{18}{x^2 - 2x - 15} \leq 0$



37) $\frac{-14}{(x-5)^2-2} \geq 0;$

38) $\frac{-13}{(x+1)^2-3} \geq 0;$

39) $25a - 5b + 22$, если $\frac{3a - 7b + 6}{7a - 3b + 6} = 4;$

40) $28a - 7b + 40$, если $\frac{2a - 5b + 7}{5a - 2b + 7} = 6;$

41) $11a - 7b + 21$, если $\frac{4a - 5b + 6}{5a - 4b + 6} = 3;$

42) $31a - 4b + 55$, если $\frac{a - 4b + 7}{4a - b + 7} = 8.$